Exam. Code : 103204

Subject Code : 1125

B.A./B.Sc. Semester—IV

MATHEMATICS

Paper—I

(Statics and Solid Geometry)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Do any *five* questions, selecting atleast *two* questions from each section. All questions carry equal marks.

SECTION—A

1. (a) Prove that the resultant of two forces acting at a point O along OA and OB and equal in magnitude to λ OA and μ OB, respectively is equivalent to (λ + μ) OC, where C is a point in AB such that λ CA = μ . CB.

   (b) ABC is a triangle and O a point in its plane. A force R acts along AO. Resolve R into two forces parallel to it and acting at B and C, respectively, where O is the circumcentre of the triangle.

2. (a) Prove that if any number of co-planer forces acting on a rigid body have a resultant, the algebraic sum of their moments about any point in their plane is equal to the moment of their resultant about that point.

(b) Three forces act at the corner of a square, each perpendicular to the plane of the square. Find their magnitudes if the resultant is a given force of magnitude R acting at the fourth corner.

3. (a) Prove that two couples acting in the same plane are equivalent to a single couple whose moment is the algebraic sum of the moments of the separate couples.

(b) Prove that any force is equivalent to an equal and parallel force at an arbitrary point together with a couple of moments equal to the moment of the given force about that point.

4. (a) One end of uniform rod is attached to a hinge and other end is supported by a string attached to the extremity of the rod. The rod and the string are inclined at the same angle θ to the horizontal, if W be the weight of the rod, show that action at the hinge is $\frac{W}{4}\sqrt{9+\cot^2\theta}$. Also find the tension in the string.

(b) A weight W is supported by friction on a plane inclined at an angle α to the horizon. Show that it can not be moved up the plane by the horizontal force less than W tan 2α.

5. (a) A uniform quadrilateral ABCD is such that the diagonal AC bisects it and BD divides it in two parts in the ratio 2 : 1. Show that it's C.G. divides AC in the ratio 5 : 4.

(b) Find C.G. of a hollow hemisphere.

SECTION—B

6. (a) A is a point on OX and B on OY so that the angle OAB is constant (= α). On AB as diameter a circle is described whose plane is parallel to OZ. Prove that as AB varies the circle generates the cone $2xy - z^2 \sin 2\alpha = 0$.

(b) Find the equation of the cone of revolution with vertex at the origin, the axis as the y-axis and semi vertical angle 30°.

7. (a) Find the equation of the cone whose vertex is (2, –3, 1) and whose guiding curve is $4x^2 + y^2 = 1$, $z = 0$.

(b) Prove that the equation $x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z - 20 = 0$ represents a cone, find its vertex.

8. (a) Prove that the equation $\sqrt{fx} + \sqrt{gy} + \sqrt{hz} = 0$ represents a cone which touches the co-ordinates planes and that the equation of the reciprocal cone is $fyz + gzx + hxy = 0$.

(b) If $\frac{x}{5}=\frac{y}{-4}=\frac{z}{1}$ is one of the sets of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, find the equations of the other two.

9. (a) Find the equation of the right circular cylinder whose axis is $\frac{x-2}{2}=\frac{y-1}{1}=\frac{z}{3}$ and passes through (0, 0, 3).

(b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 0$.

10. (a) Find the equation of the quadric cylinder with generators parallel to x-axis and passing through the curve $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = p$.

(b) Find the equation of the right circular cylinder of radius 4 and whose axis is the line $x = 2y = - z$.